

A problem generalized from a puzzle "Habits of a teacher "

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A Problem of "a habit of a teacher".

Problem 1. A teacher is going to give students a test. The test consists of 11 questions and along with each question there is an answer. The answer may be correct or false. The students are expected to write "correct" if they think the answer is correct and "false" if they think it false. The teacher has two habits. One is that he never present three correct answers in a row nor three false answers in a row. Another one is that he presents more correct ones than false ones. What is the best strategy for students if they do not know anything about the subject.

Problem 2. In the previous problem the number of questions is 11. Can you change the number of questions and find a pattern of the best strategy?

In fact there is a very interesting pattern.

Answer to Problem 1.

We need to present all the possible answers of the teacher. In fact there are 144 answers which satisfy the habit of the teacher.

The followings are all the possible answers of the teacher. In the following list we assume that 0 stands for false and 1 for true.

```
{0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1}, {0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1},
{0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1}, {0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1},
{0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1}, {0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1},
{0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1}, {0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1},
{0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0}, {0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1},
{0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1}, {0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1},
{0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 1}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1},
{0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1}, {0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1},
{0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1}, {0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0},
{0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1}, {0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1},
```



```

{1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0}, {1, 1, 0, 1, 0, 1, 0, 1, 0, 0, 1},
{1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1},
{1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0}, {1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1},
{1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0}, {1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1},
{1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0}, {1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1},
{1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0}, {1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1},
{1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0}, {1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1},
{1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0}, {1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1},
{1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0}, {1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1},
{1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0}, {1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1},

{1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0}, {1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1},
{1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0}, {1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1}

```

If we count the number of 1 (which stands for true) for 11 questions, then we have

{98, 83, 67, 83, 80, 74, 80, 83, 67, 83, 98}. By this list of numbers we know that there are 98 true answer out of 144 for the first question, 83 true answer out of 144 for the second question, ..., 98 true answer out of 144 for the last questions.

We divide this list {98, 83, 67, 83, 80, 74, 80, 83, 67, 83, 98} with 144, and get the following list of numbers {0.680556, 0.576389, 0.465278, 0.576389, 0.555556, 0.513889, 0.555556, 0.576389, 0.465278, 0.576389, 0.680556}.

Then by this list of numbers we can make the best strategy. As to the first question the percentage of 1 (which stands for true) is 68% (which is bigger than 50%) . Therefore it is better for us to write true. As to the second question percentage is 57%, and hence we write true. As to the third question percentage is 46% (which is less than 50%). Therefore we write false.

In this way we can make the best strategy for this problem.

The best strategy is {1,1,0,1,1,1,1,1,0,1,1}. In other word {True,True,False,-True,True,True,True,True,False,True,True}.

Answer to Problem 2.

If we consider the problem of n=15, the best strategy is {1,1,0,1,1,1,1,1,1,1,1,0,1,1}. In other words {True,True,False,True,True,True,-True,True,True,True,True,False,True,True}.

There is certainly a pattern!

In fact in his preprint Kohno [2] he prove that for any natural number n the best strategy of this problem is {True,True,False,True,...,True,False,True,True}. Note that you have to write only 2 "False".

Unfortunately his proof is more than 40 pages long, and not easy to understand. If you are interested in his proof, please contact Mr.Susumu Kohno at Osaka University.

Historical background for this problem . In Hujimura and Tamura [1] they introduced the 5 question version of our puzzle. The answer to this puzzle is quite simple; {True,True,False,True,True}, but when we began to study the case with more than 5 questions, a very interesting feature of the problem suddenly appeared in front of our eyes.

Unfortunately the proof of the n -question version of this problem is very difficult, and the proof is very long.

Mathematica code for the calculation . We used mathematics software *Mathematica* to find the best strategy. The following function `Ba[n]` returns all the possible answers of the teacher and the best strategy. Note that `Ba[n]` will spend much time and memory when $n > 15$. You can make your own program to calculate with your favorite computer language.

```
<< DiscreteMath`Combinatorica`

Clear[a1];
Ba[n_] := Block[
{a2, a3, a4, a5, a6, a7, a8, a9, a10, a11, a12, a13, g, h, i, j, k},
  a3 = Map[FromDigits, Strings[{2, 1}, n]];
  g[x_] := ToString[x];
  a4 = Map[g, a3];
  a5 = Select[a4, StringMatchQ[#, "*222*" ] &];
  a6 = Select[a4, StringMatchQ[#, "*111*" ] &];
  a7 = Complement[a4, Union[a5, a6]];
  h[x_] := ToExpression[x];
  a8 = Map[h, a7];
  i[x_] := IntegerDigits[x];
  a9 = Map[i, a8];
  a10 = Select[a9, Count[#, 2] > n/2 &];
  Print[Length[a10]];
  j[x_] := x - Table[1, {m, 1, n}];
  a11 = Map[j, a10];
  Print["If 0 stands for false and 1 for true, all the possible answers are"];
  Print[a11];
  a120 = Apply[Plus, a11];
  a12 = Apply[Plus, a11] / Length[a11] // N;
  Print[a120];
  Print[a12];
  a13 = Table[1, {m, 1, n}] - a12;
  Map[Round, a12]
]
```

- [1] K.Hujimura and S.Tamura: An introduction to mathematics of puzzles, Kodansha(In Japanese).
- [2] K.Kohno: An interesting feature of a certain pattern. Preprint.